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Phase signature of topological transition in Josephson Junctions

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Topological superconductivity hosts exotic quasi-particle excitations including Majorana bound states which hold promise for fault-tolerant quantum computing. The theory predicts emergence of Majorana bound states is accompanied by a topological phase transition. We show experimentally in epitaxial Al/InAs Josephson junctions a transition between trivial and topological superconductivity. We observe a minimum of the critical current at the topological transition, indicating a closing and reopening of the supercondcuting gap induced in InAs, with increasing magnetic field. By embedding the Josephson junction in a phase-sensitive loop geometry, we measure a π -jump in the superconducting phase across the junction when the system is driven through the topological transition. These findings reveal a versatile two-dimensional platform for scalable topological quantum computing.

Majorana bound states (MBS), which are their own antiparticles, are predicted to emerge as zero-energy modes localized at the boundary between a topological superconductor and a topologically-trivial region [1]. MBS can nonlocally store quantum information and their non-Abelian exchange statistics allows for the implementation of quantum gates through braiding operations [2]. This makes them ideal candidates for robust qubits in fault-tolerant topological quantum computing [3]. Rather than seeking elusive spinless p-wave superconductors required for MBS, a common approach is to use conventional s-wave superconductors to proximity-modify semiconductor heterostructures with the suitable symmetries [4].

Early proposals to realize MBS were focused on one-dimensional (1D) systems such as proximitized nanowires and atomic chains [5–9], where the observation of a quantized zero-bias conductance peak [10] provided the support for MBS. However, the inherent difficulties in the technological implementation of the required networks, together with the intrinsic instabilities of their 1D elements, have motivated the search for versatile 2D platforms exploiting the use of more conventional devices such as Josephson junctions (JJs) and spin valves [11–14]. Recent experiments [15–17] suggest that planar JJs are particularly promising because they support the transition to topological superconductivity over a large range of external parameters without requiring fine tuning.

In this work, we observe a minimum of the critical current in a JJ with increasing parallel (in-plane) magnetic field, \mathbf{B}_{\parallel} , which indicates a closing and reopening of the superconducting gap. This minimum is accompanied by a π -jump in the superconducting phase across the junction. This only occurs when \mathbf{B}_{\parallel} is applied perpendicular to the current direction. Both signatures, in a material with spin-orbit coupling (SOC), suggest that the gap that opens at high \mathbf{B}_{\parallel} is topological in nature. Theoretical simulations provide additional support for the presence of a topological transition and its compatibility with the emergence of MBS. Details of the model used are presented in Supplementary Materials. The topological phase appears resilient to small field misalignments, demonstrating potential for more complicated geometries and MBS braiding. In addition to \mathbf{B}_{\parallel} amplitude, the top gate voltage is demonstrated to be an efficient control knob for manipulating the topological phase transition.

We investigate planar JJs based on epitaxial Al/InAs, engineered to support high-interfacial transparency and robust proximity-induced superconductivity in InAs [11]. To explore potential phase-sensitive signatures of topological superconductivity, we use two JJs to form a superconducting quantum interference device (SQUID) as shown by a SEM image in Fig. 1A. We should note that the SQUID phase-measurements were crucial for identifying unconventional superconductivity in cuprates [18]. Both junctions (1, 2) of the SQUID are W=4 μ m wide and L=100 nm long, while the area of the SQUID loop is 25 μm^2 . The two junctions show small variations in normal resistance(R_n), $R_n^1 = 102 \Omega$, $R_n^2 = 110 \Omega$ and critical current(I_c) $I_c^1 = 4.4 \mu A$, $I_c^2 = 3.6 \mu A$, measured in the absence of a gate voltage. Using a vector magnet, we can apply an in-plane field along an arbitrary axis defined by θ as indicated in Fig. 1A and phase-bias our device. The versatility of our setup can be seen from the measured



Fig. 1: Physical system (A) SEM image (colorized) of a SQUID similar to the one presented. The device is composed of two 4 μ m wide JJ with a gap of 100 nm. The central area is about 25 μ m² and each junction is independently gateable. The x direction is taken colinear to the current flow in the junctions. (B) Measurement of the junction resistance at zero flux as a function of the gate voltage applied on JJ2. At $V_g^2 = 0$ V both junctions can carry a supercurrent, below $V_g^2 < -5.5$ V, JJ2 behaves like an open circuit. (C) Predicted critical current of a junction in the presence of an in-plane field along y. Above the first dashed line, the superconducting state goes from s to p-type and goes back to s-type above the second dashed line.

SQUID resistance as a function of an applied bias current, I, and V_g^2 in Fig. 1B. The critical current, at which the SQUID acquires a finite resistance, decreases and becomes constant for $V_g^2 < -5.5$ V, indicating that JJ2 is fully depleted. This shows that each JJ can be studied individually. Additional data demonstrating that we can operate this device either as a SQUID or as a single JJ are presented in Supplementary Materials.

In the presence of $\mathbf{B}_{\parallel} \equiv B_y$, theory predicts a single junction will undergo a topological phase transition. One signature of this transition is the closing (or partial closing) and re-opening of the superconducting gap as illustrated in the tight-binding simulation results presented in Fig. 1C. Above that closing, we expect the superconducting state to have transitioned towards a topological phase dominated by chiral p-type superconductivity.

Our characterization of JJs in \mathbf{B}_{\parallel} shows that both have the same critical magnetic field $B_c \approx 1.45$ T, for thin-film Al, and independent of the \mathbf{B}_{\parallel} direction. In contrast, both junctions show strong anisotropy of I_c with \mathbf{B}_{\parallel} direction as shown in Supplementary Materials. These findings are consistent with the previous measurements on JJs based on InAs 2DEG [12]. In Fig. 2A and B, we present the dependence of the critical current of JJ1 as a function of B_y at two different gate voltages: (A) $V_g^1 = -1.5$ V and (B) $V_g^1 = 1.4$ V. In Fig. 2A, at lower V_g^1 and thus at a lower density, we observe a trivial monotonic decrease of I_c with B_y . Remarkably, at higher V_g^1 in Fig. 2B we see a striking difference where the superconducting gap closes and reopens around $B_y = 600$ mT, in agreement with the tight-binding results from Fig. 1C. Above that gap closing, we measure $I_c \sim 20$ nA, consistent with the gap reopening and topological transition. A similar non-monotonic gap dependence with \mathbf{B}_{\parallel} was recently reported in HgTe [19] and InSb [17].

By considering Thouless energy, $E_T = (\pi/2)v_F/L$, where v_F is the Fermi velocity and L is the gap of the junction, one may expect to reach the topological phase more easily at low density (smaller gate voltages) since the transition has been predicted to occur around $E_Z \sim E_T$, where E_Z is the Zeeman energy [20]. However, this neglects the v_F -mismatch between the Al and InAs regions [21], and SOC [22], which can both change with density [23].

The nontrivial evolution of the superconducting gap and topological transition show similar behaviour in both junctions (JJ1 and JJ2). Figure 2C presents the zero-bias resistance of JJ2 as a function V_g^2 and B_y . At the largest V_g^2 , the transition occurs at ~ 500 mT and moves towards higher B_y , as V_g^2 is decreased. Below $V_g^2 = -1.5$ V no evidence of any transition remains. The lower magnetic field transition in JJ2 compared to JJ1 can be attributed to small variation of junction properties for example lower supercurrent and corresponding induced gap.

While the observed non-monotonic dependence of I_c with B_y is consistent with a transition to topological superconductivity, phase-sensitive measurements with a SQUID could independently confirm this scenario. However, it is generally difficult to avoid arbitrary field offsets between measurements. Here, following the approach



Fig. 2: Re-opening of superconducting gap in magnetic field Measurement of the resistance of JJ1 as function of an applied in-plane field along the y-axis at two different gate voltages (A) $V_g^1 = -1.5$ V, (B) $V_g^1 = 1.4$ V. In both cases, JJ2 is depleted ($V_g^2 = -7$ V) and does not participate in the transport. At high gate (B), a closing and re-opening of the superconducting gap is observed around 600 mT for JJ1. (C) Zero-bias resistance of JJ2 as a function of the applied in-plane field and the gate voltage. At low gate, the superconducting gap remains open up to about 1T. At higher gates a gap closing and re-opening characterized by a peak in resistance appears and moves to lower fields as the gate increases. V_q^1 is set to -7V.

described in [24], we use the gate tunability of our device to measure the phase offset between the oscillations observed at different gate voltage but acquired during a single B_z sweep. Using SQUID interferometry, we can identify topological transition by setting JJ1 at $V_g^1 = -2$ V as the reference junction. At this gate voltage, JJ1 does not show a topological transition at any field B_y . The resulting SQUID oscillations in JJ2 reveal some crucial differences between $B_y = 100$ mT and 850 mT, shown respectively in Figs. 3A and 3B, for various V_g^2 . From the results in Fig. 2C, we expect that JJ2 would never reach the topological regime at 100 mT. Indeed, in Fig. 3A, we only observe a small phase-shift which we attribute to the spin-galvanic effect, discussed in [24]. At higher B_y in Fig. 3B there is a larger phase-shift between $V_g^2 = -3$ V and -4 V than in Fig. 3A, consistent with the linear increase of spin-galvanic effect in B_y . However, comparing $V_g^2 = -1$ V and higher gate values, one can note that a phase-shift of about π occurs. This independently supports the transition to the topological phase shown for the same parameter range in Fig. 2C.

Our tight-binding calculations, presented in Fig. 3C, reveal that such a nearly π -jump in the superconducting phase is indeed a fingerprint expected for the topological transition with the emergence of MBS, shown in Supplementary Materials. Consistent with previous findings [15, 20], our simulation predicts a phase-dependent position of the gap closing in a phase-biased system, as indicted by the purple line in Fig. 3C. In Fig. 3D we present the phase-shift between the reference scan performed at $V_g^2 = -4$ V and subsequent gate values. At low B_y , below the topological transition, we observe that the phase is linear in B_y , as indicated by the solid lines corresponding to linear fits to the values below 450 mT. The increase in slope with V_g^2 can be attributed to the increase of SOC [24]. For $V_g^2 = -3$ V and -1 V, the linear trend holds up over all B_y . However, for $V_g^2 = 1$ V, 2 V and 3 V, a jump can be observed around 550 mT, followed by another linear portion. To separate these effects, we first unwrap the phase and then subtract the linear component extracted from low- B_y fits. The re-plotted results in Fig. 3E reveal near π phase-jump around the observed topological transition, consistent with theoretical predictions.

A distinct feature of the observed topological transition is its interplay of SOC and \mathbf{B}_{\parallel} . In our material the topological regime is expected when \mathbf{B}_{\parallel} is along the y-direction, i.e. $\theta = 0$. We test this by probing the gap



Fig. 3: Phase signature of topological transition from SQUID interferometry (A-B) SQUID oscillations for $B_y = 100 \text{ mT}$ (A) and 850 mT (B) for different V_g^2 and $V_g^1 = -2 \text{ V}$. The dashed lines indicate the position of the maximum at $V_g^2 = -4 \text{ V}$ used as a phase reference. The stars mark the position of the maximum of the oscillation. The solid orange lines are best fits to the SQUID oscillations used to extract the period and the phase shift between different V_g^2 values. (C) Ground-state phase (red solid line) and calculated SQUID phase-shift (blue and green dashed lines), see Supplementary Materials, as a function of the in-plane field calculated using a tight-binding model. A phase jump of about π occurs at the field corresponding to the closing of the gap identified in Fig. 1C. Purple lines indicate the phase and field at which the zero-temperature energy gap closes in a phase-biased system. (D) Phase difference between the SQUID oscillation at $V_g^2 = -4 \text{ V}$ and the oscillation at a different value as a function of B_y . The solid lines correspond to linear fits of the data for $B_y \leq 450 \text{ mT}$. (E) Phase shift from which the linear B_y -contribution has been subtracted to highlight the phase jump occurring for the two higher V_g^2 values.

closing in a tilted \mathbf{B}_{\parallel} , away from B_y . In Fig. 4 we show zoom-ins of the gap-closing of JJ1 at $V_g^1 = 1.4$ V at different angles. As θ is increased, the closing of the gap is weakened. Similarly, SQUID data at $\theta = 10^\circ$, shown in Supplementary Materials, display a reduced phase-shift, which may indicate a reduced topological gap. Unlike at smaller angles, at $\theta = 20^\circ$ the gap decreases monotonically which suggests that s-wave order prevails and no transition is observed.

In conclusion, we have presented a study of the closing and re-opening of the superconducting gap in Josephson junctions fabricated on Al/InAs. By embedding the junction in a SQUID loop, we are able to measure the π -jump that accompanies the re-opening of the gap. These findings strongly supports the emergence of a topological phase in the system. This offers a scalable platform for detection and manipulation of Majorana bounds states for development of complex circuits for fault-tolerant topological quantum computing. The versatility of this two-dimensional geometry and SQUID manipulation may also support other exotic phases probed by phase-sensitive signatures [25].

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Fig. 4: In-plane magnetic-field anisotropy of the gap closing. Resistance of JJ1 as function of the bias current and the y-component of \mathbf{B}_{\parallel} , applied at an angle θ with respect to the y-direction as depicted in Fig. 1A.

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Supplementary materials

I. MATERIALS AND METHODS

A. Growth and fabrication

The Josephson junction (JJ) structure is grown on semi-insulating InP (100) substrate. This is followed by a graded buffer layer. The quantum well consists of a 4 nm layer of InAs grown on a 6 nm layer of $In_{0.81}Ga_{0.25}As$. The InAs layer is capped by a 10 nm $In_{0.81}Ga_{0.25}As$ layer which has been found to produce an optimal interface while maintaining high 2DEG mobility [23]. This is followed by *in situ* growth of epitaxial Al (111). Molecular beam epitaxy allows growth of thin films of Al where the in-plane critical field can exceed 2 T [11].

Devices are patterned by electron beam lithography using PMMA resist. Transene type D is used for wet etching of Al and a III-V wet etch $(H_2O : C_6H_8O_7 : H_3PO_4 : H_2O_2)$ is used to define deep semiconductor mesas. We deposit 50 nm of AlO_x using atomic layer deposition to isolate gate electrodes. Top gate electrodes consisting of 5 nm Ti and 70nm Au are deposited by electron beam deposition.



Fig. S1: Structure of the fabricated Josephson Junction

B. Measurements

The device has been measured in an Oxford Triton dilution refrigerator fitted with a 6-3-1.5 T vector magnet which has a base temperature of 7 mK. All transport measurements are performed using standard dc and lock-in techniques at low frequencies and excitation current $I_{ac} = 10$ nA. Measurements are taken in a current-biased configuration by measuring R=dV/dI with I_{ac} , while sweeping I_{dc} . This allows us to find the critical current at which the junction or SQUID switches from the superconducting to resistive state. It should be noted we directly measure the switching current, which can be lower than the critical current due to effects of noise. For the purposes of this study we assume they are equivalent.

II. ADDITIONAL EXPERIMENTAL RESULTS

A. Operation of the SQUID as a single junction

The device described through the paper is a SQUID whose both junctions (JJ1, JJ2) can be gated independently. In Fig. **S2**, we illustrate how we can go from a SQUID regime in which fast SQUID oscillations are clearly visible atop the Fraunhoffer pattern of the junctions (A), to a single junction regime in which SQUID oscillations are completely absent but we preserve the Fraunhoffer pattern of the junction which is not depleted (B).



Fig. S2: SQUID to single junction transition.(A) SQUID oscillations of the device when both Josephson junctions are not gated and in the absence of in-plane field. (B) Equivalent scan when JJ2 is fully depleted by applying -7 V on V_g^2 which reduces to Fraunhoffer pattern.

B. Fraunhofer pattern in the presence of a parallel magnetic field

The application of an in-plane magnetic field on the sample leads to a reduction of the critical current of the Josephson junctions and a distortion of the Fraunhoffer pattern as illustrated in Fig **S3**.



Fig. S3: Fraunhofer pattern of JJ 1 in the presence of an in-plane field. (A) Fraunhofer pattern when applying 250 mT along the x-direction i.e. parallel to the current. (B) Fraunhofer pattern when applying 500 mT along the y-direction.

The change in the critical current of the JJ appears to strongly depends on the direction of the applied in-plane field. In Fig. **S3**, the amplitude of the critical current is similar in both plots but the magnitude of the applied magnetic field is twice as large in the y direction (A) compared to the x direction (B).

For both directions of the field, the Fraunhoffer pattern appears asymmetric which is not the case in the absence of the in-plane as illustrated in the main text. The observed distortions are similar for both orientation of the field.

When comparing those data to the ones presented in the main text, one can notice that the width of the first node has been divided by about two. We attribute this effect, which is also visible in the SQUID oscillations, to the transition out of the superconducting state of the indium layer at the back of the sample. The transition occurs around 30 mT and does not impact our study otherwise.

C. Gap closing at finite magnetic field driven by gate voltage

As illustrated in Fig. 2 of the main text, one can drive the system from the trivial state to the topological state at a finite field by increasing the gate voltage resulting in both an increase of the electronic density and in an increase of spin-orbit coupling strength. In Fig. **S4**, we present the superconducting gap closing and re-opening as a function of the gate voltage applied to the junction in the presence of a parallel field of 750 mT applied along the y-axis.



Fig. S4: Gate driven topological transition. Measured JJ2 resistance as a function the applied gate voltage in the absence and presence of in-plane field of 750 mT along the y direction. The JJ 1 is depleted by applying $V_q^1 = -7$ V.

D. Phase jump across the gap closing: Magnetic field applied at $\theta = 10^{\circ}$

We observe in Fig. 4 that the partial closing of the superconducting gap survives up to an angle of $\theta \sim 10^{\circ}$, away from the y-direction. We present in Fig. **S5** the measured phase jump through the transition. While a phase jump can be observed, its magnitude is reduced compared to the perfectly aligned ($\theta = 0^{\circ}$) situation.

III. THEORETICAL CALCULATION DETAILS

Theoretical simulations of a single Josephson junction were performed by using the Bogoliubov-de Gennes Hamiltonian,

$$H = \left[\frac{p^2}{2m^*} - \mu_S + \frac{\alpha}{\hbar} \left(p_y \sigma_x - p_x \sigma_y\right) + V_0(x)\right] \tau_z - \frac{g^* \mu_B}{2} \mathbf{B} \cdot \boldsymbol{\sigma} + \Delta(x) \tau_+ + \Delta^*(x) \tau_- , \qquad (1)$$

where **p** is the momentum, μ_S the chemical potential in the S region, α is the Rashba SOC strength, **B** is the external magnetic field, and $m^* = 0.03 m_0$ and $g^* = 10$ are the electron effective mass and effective g-factor in InAs, respectively. The function $V_0(x) = (\mu_S - \mu_N)\Theta(L/2 - |x|)$ describes the changes in the N-region chemical potential (μ_N) due to the application of the gate voltage, while $\Delta(x) = \Delta e^{i \operatorname{sgn}(x)\phi/2}\Theta(|x| - W/2)$ accounts for the spatial dependence of the superconducting gap amplitude and the corresponding phase difference (ϕ). The τ -matrices are the Nambu matrices in the electron-hole space and $\tau_{\pm} = (\tau_x \pm \tau_y)/2$. The eigenvalue problem for the BdG Hamiltonian is numerically solved by using a finite-difference scheme on a discretized lattice as implemented in Kwant [26], with a lattice constant a = 10 nm. The calculated eigenenergies (E_n) are then used to compute the free energy [27, 28],

$$F = -2k_B T \sum_{E_n > 0} \ln\left[2\cosh\left(\frac{E_n}{2k_B T}\right)\right].$$
(2)



Fig. S5: Phase jump in the presence of a misaligned field (A) Phase difference between the SQUID oscillation at $V_g^2 = -4V$ and the oscillation at a different value as a function of the applied in-plane field along the y direction. The field is applied with a 10° angle with respect to y-direction. The solid lines correspond to linear fits on the data for $B_y \le 450$ mT. (B) Phase shift from which the contribution linear in the magnetic field has been subtracted to highlight the phase jump.

and the supercurrent,

$$I(\phi) = \frac{2e}{\hbar} \frac{dF}{d\phi}.$$
(3)

The ground state phase (ϕ_{GS}) is the phase that minimizes the free energy and the critical current (I_c) corresponds to the maximum of the supercurrent with respect to the phase, i.e. $I_c = \max_{\phi} I(\phi)$.

The temperature and magnetic field dependences of the superconducting gap are taken into account by using the BCS relation,

$$\Delta(T,B) \approx \Delta(T,0) \sqrt{1 - \left[\frac{B}{B_c(T)}\right]^2},\tag{4}$$

where

$$\Delta(T,0) \approx \Delta_0 \tanh\left[1.74\sqrt{\frac{T_c}{T}-1}\right].$$
(5)

Here $\Delta_0 = 1.74k_BT_c$ with T_c as the superconductor critical temperature. The temperature dependence of the critical magnetic field is approximated as,

$$B_c(T) = B_c \left[1 - \left(\frac{T}{T_c}\right)^2 \right],\tag{6}$$

where B_c is the critical magnetic field at zero temperature.



Fig. S6: Simulations results (A) Magnetic field dependence of the low-energy ground-state spectrum of a JJ. The energy gap closes and reopens at the field value for which ϕ_{GS} starts to shift from zero to nearly π (see Fig. 3C in the main text), indicating a topological phase transition and the emergence of Majorana bound states (MBS). The red lines indicate the evolution of finite-energy states into MBS inside the topological gap. (B) Probability density of the MBS in the JJ for B = 0.7 T (see black dashed line in (A)). The probability density, which has been normalized to its maximum value, clearly indicated the formation of MBS localized at the end of the junction. The green dashed lines indicate the edges of the normal region.

For the numerical simulations we used $\mu_S = \mu_N = 0.5$ meV, $\Delta_0 = 0.23$ meV, $\alpha = 10$ meV nm, and $B_c = 1.6$ T. For the calculation of the critical current, we used periodic boundary conditions along the junction and assumed $k_B T = 0.3\Delta_0$.

Figure S6A presents the magnetic field dependence of the low-energy ground-state spectrum, i.e. the spectrum calculated when the phase difference across the junction equals ϕ_{GS} and the corresponding free energy is minimized. At low field $\phi_{GS} \approx 0$ the JJ is in the topologically trivial states with no MBS. As the field increases, the energy gap closes and reopens at a field of about 0.5 T, indicating a topological phase transition in which finite-energy states evolve into MBS (red lines) residing inside the topological gap. The topological transition is accompanied by a shift in the ground-state phase from zero to a value close to π (see Fig. 3C in the main text). In Fig. S6B, we plot the normalized probability density of the lowest energy states at B = 0.7 T. The localization of these zero-energy states at the ends of the junction is a clear indication of the formation of MBS. The green dashed line marks the frontier between the middle area of the junction which is not in contact with the superconductor and the outer regions.

For the theoretical calculation of the shift phase, the SQUID supercurrent was taken as,

$$I_t = I_{c1} \frac{\sin(\phi_1 - \phi_0)}{\sqrt{1 - \tau \sin[(\phi_1 - \phi_0)/2]^2}} + I_2(\phi_2).$$
(7)

The first contribution with τ characterizing the junction transparency describes JJ1, which is kept in the topologically trivial phase, while the second contribution, describing JJ2, is numerically computed as described above. The phases ϕ_1 and ϕ_2 corresponding, respectively, to JJ1 and JJ2 are related to the flux piercing the SQUID, $\phi_1 - \phi_2 = 2\pi\Phi/\Phi_0$ (with Φ_0 as the flux quantum). The phase ϕ_0 represents the anomalous contribution linear in the in-plane magnetic field B_y . By maximizing the total current I_t with respect to ϕ_2 we obtain the flux dependence of the critical current and the corresponding phase shift and extract the linear contribution. The theoretical phase-shift is shown in Fig. 3C.

In Fig. **S7**, we present the numerical results of the impact of the field misalignment on the gap closing. The simulation appears far less sensitive to the field misalignment than what was experimentally observed since the gap



Fig. S7: Magnetic field dependence of the critical current for different orientations of the in-plane magnetic field. As the magnetic field orientation deviate from being parallel to the junction, the minimum of the critical current occurs at larger magnetic field amplitudes and eventually disappear, indicating the suppression of the topological phase transition. This trend is in qualitative agreement with the experimental observations.

reduction persists for an angle θ of 50°. We believe this discrepancy is due to size effects not accounted for in the theoretical simulations of the critical current, which assume a system 400 nm long in the x direction and infinite in the y direction, while the actual experimental dimensions are about 2 μ m along x and 4 μ m along y. Numerical simulations of the critical current for such a large system become extremely challenging.